



# COMMON PRE-BOARD EXAMINATION 2022-23



## Subject: MATHEMATICS (041) Marking Scheme

Class: XII

Time: 3 Hours

Date: \_\_\_\_\_

Max. Marks: 80

Q.No.		Marks
	<b><u>SECTION - A</u></b>	
	<b>(Section A consists of 20 questions of 1 mark each)</b>	
1.	D	1
2.	C	1
3.	B	1
4.	D	1
5.	C	1
6.	A	1
7.	B	1
8.	A	1
9.	B	1
10.	C	1
11.	A	1
12.	B	1
13.	<b>B</b>	1
14.	A	1
15.	A	1
16.	B	1
17.	A	1
18.	D	1
19.	C	1
20.	A	1

<b><u>SECTION-B</u></b> <b>(Section B consists of 5 questions of 2 marks each)</b>		
21.	<p>Cofactor of <math>-3</math> is <math>46</math>          OR  <math> A  = -3</math></p> $\text{adj}A = \begin{bmatrix} -1 & -1 & 3 \\ 2 & 5 & -12 \\ 1 & -2 & 0 \end{bmatrix}^T$ $x = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ <p>Hence, <math>X = 1, Y = 1</math> and <math>Z = 2</math></p>	$\frac{1}{2}+1/2+1$
22.	$\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}(-1).$ $= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = \frac{-2\pi + 4\pi - 3\pi}{12}$ $= \frac{-5\pi + 4\pi}{12} = \frac{-\pi}{12}$	$1+1$
23.	$\frac{dy}{dx} = x^{\sin x} \left( \frac{\sin x}{x} + \log x \cos x \right) + (\sin x)^x (x \cot x + \log \sin x)$ <p>OR</p> $\frac{dy}{dx} = \frac{y}{x} \left( \frac{x \log y - y}{y \log x - x} \right)$	$1+1$
24.	<p>Given : Here probability of A and B that can solve the same problem is given , i.e., <math>P(A) = \frac{2}{3}</math> and <math>P(B) = \frac{3}{5} \Rightarrow P(\bar{A}) = \frac{1}{3}</math> and <math>P(\bar{B}) = \frac{2}{5}</math></p> <p>Also, A and B are independent . not A and not B are independent.</p> <p>To Find: atleast one of A and B will solve the problem</p> <p>Now , <math>P(\text{atleast one of them will solve the problem}) = 1 - P(\text{both are unable to solve})</math></p> $= 1 - P(\bar{A} \cap \bar{B})$ $= 1 - P(\bar{A}) \times P(\bar{B})$ $= 1 - \left( \frac{1}{3} \times \frac{2}{5} \right)$ $= \frac{13}{15}$	$1+1$

25.	<p>Since, <math>\vec{a}</math> and <math>\vec{b}</math> are perpendicular  <math>\therefore \vec{a} \cdot \vec{b} = 0</math>  <math>\Rightarrow (\lambda\hat{i} + 2\hat{j} + \hat{k}) \cdot (4\hat{i} - 9\hat{j} + 2\hat{k}) = 0</math>  <math>\Rightarrow (\lambda)(4) + (2)(-9) + (1)(2) = 0</math>  <math>\Rightarrow 4\lambda - 18 + 2 = 0</math>  <math>\Rightarrow 4\lambda - 16 = 0</math>  <math>\Rightarrow 4\lambda = 16</math>  <math>\Rightarrow \lambda = \frac{16}{4}</math>  <math>\Rightarrow \lambda = 4</math></p>	1+1
	<p><b>SECTION-C</b>  <b>(Section C consists of 6 questions of 3 marks each)</b></p>	
26.	$\frac{-y}{\sqrt{1-y^2}} dy = xe^x dx$ <p>Therefore, on integrating both sides, we get,</p> $\int \frac{-y}{\sqrt{1-y^2}} dy = \int xe^x dx$ $\Rightarrow \sqrt{1-y^2} = xe^x - e^x + C \dots(i)$ <p>Also, given that <math>y = 1</math>, when <math>x = 0</math>  On putting <math>y = 1</math> and <math>x = 0</math> in Eq. (i), we get  <math>\sqrt{1-1} = 0 - e^0 + C</math>  <math>\Rightarrow C = 1 \quad [ \because e^0 = 1 ]</math></p> <p>On substituting the value of <math>C</math> in Eq. (i), we get  <math>\sqrt{1-y^2} = xe^x - e^x + 1</math></p>	1+1+1
27.	$\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$ $I = \int_0^{\frac{\pi}{4}} \log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right) dx$ $= \int_0^{\frac{\pi}{4}} \log\left[\frac{2}{1+\tan x}\right] dx$ $I = \int_0^{\frac{\pi}{4}} \log 2 dx - \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$ $I = \frac{\pi}{8} \log 2$	1+1+1
28.	$I = \int \left(1 + \frac{2x+1}{x^2+3x+2}\right) dx$	1+1+1

	$I = x + \log x^2 + 3x + 2  - 2 \log\left \frac{x+1}{x+2}\right  + C$ OR $I = \frac{3}{5}\log(x+2) + \frac{1}{5}\log(x^2 + 1) + \frac{1}{5}\tan^{-1}x + C$	
29.	$\frac{dy}{dx} = \frac{-(xy+y^2)}{x^2}$ Put $y = vx$ $\log \frac{vx^2}{v+2} = \log C$ $\frac{yx^2}{y+2x} = C$ $3yx^2 = y + 2x$ OR $\sin\left(\frac{y}{x}\right) = \log Cx$	1+1+1
30.	Graph, shaded region Maximum value is Rs 4000 at C (4,4)	1+1+1
31.	Prove reflexive, symmetric, transitive, equivalence relation	1+1+1
	<b><u>SECTION-D</u></b> <b>(Section D consists of 4 questions of 5 marks each)</b>	
32.	Figure Point of intersection are (4,12) and (-2,3) Area = 27 sq units	1+2+1+1
33.	$\int_1^4 (x-1)dx - \int_1^2 (x-2)dx + \int_2^4 (x-2)dx - \int_1^3 (x-3)dx + \int_1^4 (x-3)dx$ $= \frac{19}{2}$ OR $\frac{11}{4}$	1+1+1+1+1
34.	$Sd = \frac{ (\vec{a} - \vec{a}_1) \cdot \vec{b}_1 \times \vec{b}_2 }{\vec{b}_1 \times \vec{b}_2} = \frac{8}{\sqrt{29}}$ OR $SD=0$	1+1+1+1+1
35.	Find $\frac{dy}{dx}, \frac{d^2y}{dx^2}$	1+1+1+1+1
	<b><u>SECTION-E</u></b> <b>(Case study based questions are compulsory)</b>	

36.

(i) Since 'C' is cost of making tank

$$\therefore C = 70xy + 45 \times 2(2x + 2y)$$

$$\Rightarrow C = 70xy + 90(2x + 2y)$$

$$\Rightarrow C = 70xy + 180(x + y) [\because 2 \cdot x \cdot y = 8 \Rightarrow y = \frac{8}{2x} \Rightarrow y = \frac{4}{x}]$$

$$\Rightarrow C = 70x \times \frac{4}{x} + 180\left(x + \frac{4}{x}\right)$$

$$\Rightarrow C = 280 + 180\left(x + \frac{4}{x}\right)$$

4

(ii)  $x \cdot y = 4$ Volume of tank = length  $\times$  breadth  $\times$  height (Depth)

$$8 = x \cdot y \cdot 2$$

$$\Rightarrow 2xy = 8 \Rightarrow xy = 4$$

(iii) For maximum or minimum

$$\frac{dC}{dx} = 0$$

$$\frac{d}{dx}(280 + 180\left(x + \frac{4}{x}\right)) = 0 \Rightarrow 180\left(1 + 4\left(-\frac{1}{x^2}\right)\right) = 0$$

$$\Rightarrow 180\left(1 - \frac{4}{x^2}\right) = 0 \Rightarrow 1 - \frac{4}{x^2} = 0$$

$$\Rightarrow \frac{4}{x^2} = 1 \Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

$\Rightarrow x = 2$  (length can never be negative)

OR

$$\text{Now, } \frac{d^2C}{dx^2} = 180\left(+\frac{8}{x^3}\right)$$

$$\Rightarrow \frac{d^2C}{dx^2} \Big|_{x=2} = 180 \times \frac{8}{8} = 180 = +ve$$

Hence, to minimize C,  $x = 2$ m

37.

4

- (i)  $\therefore$  Required matrix, say P, is given by  
 $Hatchback \quad Sedan \quad SUV$

$$P = \begin{matrix} A & \begin{bmatrix} 120 & 50 & 10 \end{bmatrix} \\ B & \begin{bmatrix} 100 & 30 & 5 \end{bmatrix} \\ C & \begin{bmatrix} 90 & 40 & 2 \end{bmatrix} \end{matrix}$$

In 2020, dealer A sold 300 Hatchbacks, 150 Sedans, 20 SUVs dealer B sold 200 Hatchbacks, 50 sedans, 6 SUVs dealer C sold 100 Hatchbacks, 60 sedans, 5 SUVs.

- $\therefore$  Required matrix, say Q, is given by  
 $Hatchback \quad Sedan \quad SUV$

$$Q = \begin{matrix} A & \begin{bmatrix} 300 & 150 & 20 \end{bmatrix} \\ B & \begin{bmatrix} 200 & 50 & 6 \end{bmatrix} \\ C & \begin{bmatrix} 100 & 60 & 5 \end{bmatrix} \end{matrix}$$

- (ii)  $Hatchback \quad Sedan \quad SUV$

$$\begin{matrix} A & \begin{bmatrix} 300 & 150 & 20 \end{bmatrix} \\ B & \begin{bmatrix} 200 & 50 & 6 \end{bmatrix} \\ C & \begin{bmatrix} 100 & 60 & 5 \end{bmatrix} \end{matrix}$$

In 2020, dealer A sold 300 Hatchback, 150 Sedan, 20 SUV dealer B sold 200 Hatchback, 50 sedan, 6 SUV dealer C sold 100 Hatchback, 60 sedan, 5 SUV.

- $\therefore$  Required matrix, say Q, is given by  
 $Hatchback \quad Sedan \quad SUV$

$$Q = \begin{matrix} A & \begin{bmatrix} 300 & 150 & 20 \end{bmatrix} \\ B & \begin{bmatrix} 200 & 50 & 6 \end{bmatrix} \\ C & \begin{bmatrix} 100 & 60 & 5 \end{bmatrix} \end{matrix}$$

- (iii) Total number of cars sold in two given years, by each dealer, is given by

$Hatchback \quad Sedan \quad SUV$

$$P + Q = \begin{matrix} A & \begin{bmatrix} 120 + 300 & 50 + 150 & 10 + 20 \end{bmatrix} \\ B & \begin{bmatrix} 100 + 200 & 30 + 50 & 5 + 6 \end{bmatrix} \\ C & \begin{bmatrix} 90 + 100 & 40 + 60 & 2 + 5 \end{bmatrix} \end{matrix}$$

Hatchback Sedan SUV

$$A = \begin{bmatrix} 420 & 200 & 30 \\ 300 & 80 & 11 \\ 190 & 100 & 7 \end{bmatrix}$$

OR

The amount of profit in 2020 received by each dealer is given by the matrix

Hatchback Sedan SUV

$$A = \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix} \quad \begin{bmatrix} 50000 \\ 100000 \\ 200000 \end{bmatrix}$$

$$B = \begin{bmatrix} 15000000 + 15000000 + 4000000 \\ 10000000 + 5000000 + 1200000 \\ 5000000 + 6000000 + 1000000 \end{bmatrix}$$

$$C = \begin{bmatrix} 34000000 \\ 16200000 \\ 12000000 \end{bmatrix}$$

38. (i) Required probability =  $P(\text{one ticket with prime number and other ticket with a multiple of 4})$

$$= 2 \left( \frac{15}{50} \times \frac{12}{49} \right) = \frac{36}{245}$$

- (ii)  $P(\text{First ticket shows an even number and second ticket shows an odd number})$

$$= \frac{25}{50} \times \frac{25}{49} = \frac{25}{98}$$

4

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